Estimation of I/Q Imbalance in MIMO OFDM

Kanchan Nikumbh

Achala Deshmukh

Abstract—The estimation of in-phase and quadrature-phase (I/Q) imbalance for multiple-input multiple output (MIMO) orthogonal frequency division multiplexing (OFDM) systems using training sequences is studied in this paper. A new concept called channel residual energy (CRE) is introduced. By minimizing the CRE, we can estimate the I/Q imbalance using channel response. The proposed method needs only one OFDM block for training and the training symbols can be arbitrary. Simulation results show that the mean-squared error (MSE) of the proposed method is close to the Cramer-Rao bound (CRB).

Key Words - MIMO OFDM, I/Q imbalance.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM)-based physical layers have been selected for several wireless standards such as IEEE 802.11a, IEEE 802.11g, IEEE P802.15.3, IEEE 802.20, and IEEE 802.16. A low-cost implementation of such physical layers is challenging in view of impairments associated with the analog components. One such impairment is the imbalance between the I and Q branches when the received radio-frequency (RF) signal is down-converted to baseband. A problem with direct conversion receivers when compared to heterodyne receivers is that the baseband signals are more severely distorted by imbalances within the I and Q branches.

The direct conversion receiver has drawn a lot of attention due to its low power consumption and low implementation cost. However some mismatches in direct conversion receiver can seriously degrade the system performance, such as in-phase and quadrature-phase (I/Q) imbalance and carrier frequency offset (CFO). The I/Q imbalance is due to the amplitude and phase mismatches between the I and Q branch of the local oscillator whereas the CFO is due to the mismatch of carrier frequency at the transmitter and receiver..It is known that the I/Q imbalance and CFO can cause a serious inter-carrier interference (ICI)

in OFDM [1]. The fact that the size of the DFT matrix is usually larger than the channel length in OFDM systems, a time-domain method was proposed for the joint estimation of I/Q and channel response [2]. Both the frequency-domain and time-domain methods need only one OFDM block for training and can achieve a good performance. A lot of interests is developed in combining the OFDM systems with the multiple-input multiple-output (MIMO) technique. These systems are known as MIMO-OFDM systems. Many methods have been proposed to deal with the channel estimation in MIMO OFDM systems. One of these methods is to send the training sequences (known to the receiver) from the transmitter. The design of training sequences for MIMO OFDM systems was investigated in [5][8]. A special class of the optimal training sequences in one OFDM block is derived. However these methods assume that there is no mismatch of the local oscillators. Several methods have been developed for the estimation of the I/Q and CFO mismatches [7][9]. The compensation of I/Q imbalance for MIMO OFDM systems was investigated in [9]. The I/Q compensation method is applied in [9] for MIMO OFDM systems. The combined MMSE and MLD decoders is used for the I/Q compensation.

A. OFDM

OFDM is a multi channel modulating technique that makes use of Frequency Division Multiplexing (FDM) of orthogonal multi carriers being modulated by a low bit rate digital stream. In FDM, inter channel interference is diminished by the deterrence of the spectral overlapping of sub-carriers but it guides to an inadequate use of spectrum. To prevail over this obstacle OFDM uses orthogonal sub-carrier that helps an efficient use of the spectrum. This can be achieved by spacing the channels much closer to each other as shown in Fig(1).

The orthogonality state shows that each sub carrier contains exactly integer number of cycles in the interval period.

Some of the good reasons that OFDM become more popular in the wireless industry today are:
• Emitter and receiver are efficiently implemented with FFT/IFFT.
• Throughput maximization.
• Effectiveness against channel distortion Multi-path delay spread tolerance.

Even though OFDM has numerous advantages it has some disadvantages on the quality of the analogue radio frequency front end of both transmitter and receiver:
• Sensitive to carrier frequency errors.
• To maintain the orthogonality between subcarriers, the amplifiers need to be linear.

OFDM systems have high peak-to average ratio which may require large amplifier.

B. MIMO

In MIMO system, a number of antennas are placed at the transmitting and receiving ends, their distances are separated far enough. It is one of several forms of smart antenna technology. MIMO technology has attracted attention in wireless communications, because it offers significant increases in data throughput and link range without additional bandwidth or increased transmit power. It achieves this goal by spreading the same total transmit power over the antennas to achieve an array gain that improves the spectral efficiency (more bits per second per hertz of bandwidth) or to achieve a diversity gain that improves the link reliability (reduced fading). The MIMO system is shown in Fig.2.

C. MIMO-OFDM

At the transmitting end, a number of transmission antennas are used. An input data bit stream is supplied into space-time coding, then modulated by OFDM and finally fed to antennas for sending out radiation. At the receiving end, incoming signals are fed into a signal detector and processed before recovery of the original signal. Fig. 3 shows the basic structure of a MIMO-OFDM system.

D. I/Q Mismatch

OFDM systems are susceptible to analog front end imperfections; IQ imbalance is one of the impairments that cause the received symbols not to be correctly demodulated. Because of this front end imperfection the analog part is the expensive one in the system.

Down conversion is a basic phase in all radio frequency (RF) architectures. To convert RF signal into its equivalent baseband many configuration are used. The traditional OFDM system employs the superheterodyne receiver to convert RF signal into baseband and vice versa. This architecture use intermediate frequencies (IF) to convert RF signal down to baseband. The disadvantage of this configuration is it uses a lot of components which are expensive and external to get a good signal quality.

II. SYSTEM DESCRIPTION

Fig.3. shows an MIMO OFDM system where the numbers of transmit and receive antenna are Nt and Nr respectively. The input vector S is an Mx1 vector containing the modulation symbols. After taking the M-point IDFT of S, we obtain the M x 1 vector X. After the insertion of a CP of length L – 1, the signal is transmitted from the jth transmit antenna. Let the channel impulse response from the jth transmit antenna to the kth receive antenna be h_{kj}(n). Assuming that the lengths of all the channels are ≤ L and the length of the cyclic prefix (CP) is L – 1. So there is no inter block interference between adjacent OFDM blocks after CP removal. The received vector at the kth receive antenna can be written as

\[ r_k = \left[ H_{k,0} H_{k,1} \ldots \ldots H_{k,N-1} \right]^T x_0 x_1 \ldots x_{N-1} + q_k \] (1)

The vector due to CFO is

\[ y_k = E_k R_k \] (2)

The received vector due to I/Q mismatch becomes
\[ z_k = \mu_k y_k + v_k y_k^* \]  
\[ \text{Substituting (2) into (3), we get} \]
\[ z_k = \mu_k E_k r_k + v_k E_k^* r_k^* \]  
\[ \text{If } \theta_k \text{ is also known at the receiver, from 2.2 we can recover} \]
a scaled version of the desired baseband vector by
\[ \mu_k y_k = E_k^* \mu_k y_k \]  
\[ \text{(5)} \]

### III. PROPOSED SYSTEM

The block diagram consist of OFDM system in MIMO in which I/Q is introduced as shown in Fig.4. For proposed system we will calculate I and Q from which CRE (Channel Residual Energy) is estimated and MSE is analyzed. Thus original OFDM signal is obtained.

#### A. Estimation of I/Q Imbalance

Assume that there is no CFO. Hence we have \( \theta = 0 \) and \( E = I \). From (2.5), \( \alpha \) is related to the received vector \( z \) as
\[ \mu r = \frac{z - \alpha c^*}{1 - \alpha^2} \]  
\[ \text{(6)} \]

From (6), if \( \alpha \) is given, an estimate of the MIMO channel response can be obtained as
\[ \tilde{\mu} = B^{-1} \mu r \]  
\[ \text{(7)} \]

Where
\[ B \equiv \begin{bmatrix} A_0^T A_0 & \cdots & A_{N-1}^T A_{N-1} \end{bmatrix} \]
\[ \tilde{d} = \begin{bmatrix} d_0^T & \cdots & d_{N-1}^T \end{bmatrix} \]  

When \( \alpha \) is estimated perfectly, the first \( L \) entries of each \( c_{h_j} \) in the above expression will give us an estimate of the channel response and the last \( (\rho-L) \) entries of \( c_{d_j} \) are solely due to the channel noise. For moderately high SNR, the energy of these entries should be small. Let us define a quantity called the channel residual energy (CRE) as
\[ \text{CRE} = \sum_{l=0}^{N-1} \sum_{i=1}^{\rho-1} |\mu d_i|^2 \]  
\[ \text{(8)} \]

where \( |\mu d_i|^2 \) denotes the \( i^{th} \) entry of \( \mu\cdot d \). Any error in the estimation of \( \alpha \) will increase the CRE. Based on this observation, by minimizing the CRE we are able to estimate the I/Q parameter \( \alpha \) without knowing the channel response. To do this, we first define the \((\rho - N\alpha) \times \rho \) matrix
\[ P = \begin{bmatrix} 0 \ldots 0 \end{bmatrix} \]
\[ \text{(9)} \]

Suppose that \( \rho > L \) so that \( P \) is not a zero matrix. Multiplying \( \mu d \) by \( P \), we can rewrite the CRE as
\[ \text{CRE} = \|P \mu \tilde{d}\|^2 = \|PB^{-1}(z - \alpha c^*)\|^2 \]  
\[ \text{(10)} \]

Goal is to find \( \alpha \) that minimizes the CRE. Since for most applications, \( \alpha \) is small, (3.7) can be approximated as
\[ \text{CRE} = \|PB^{-1}(z - \alpha c^*)\|^2 \]  
\[ \text{(11)} \]

From linear algebra, it is known that the optimal \( \alpha \) that minimizes the CRE is
\[ \alpha_{opt} = \frac{(PB^{-1}z^*)(PB^{-1}z)}{\|PB^{-1}z\|^2} \]  
\[ \text{(12)} \]

By substituting \( \alpha_{opt} \) into (7), we get the estimated MIMO channel response \( \tilde{\mu} \). For the compensation of I/Q imbalance, one can employ (6) to obtain \( \mu r \). Notice that there is no need to compensate the factor \( \rho \) because it will be canceled when we use \( \mu d \) to implement the FEQ. From (12), we see that to get \( \alpha_{opt} \), we only need to compute \( B^{-1}z \) (as \( B \) is fixed, \( B^{-1} \) can be pre-computed) and perform vector inner products at the numerator and denominator. When the training sequence in [2] is used, \( B \) becomes unitary and circulant. As \( B^{-1}z \) is also circulant and unitary, \( B^{-1}z \) can be efficiently realized using circular convolution.

#### B. AN ANALYSIS OF CRE
In the following, we assume that $\alpha$ is small so that the second order term can be ignored. Also we assume that SNR is moderately high so that $\alpha \, q$ can be ignored in the analysis below. Let the estimate of $\alpha$ and be $\tilde{\alpha}$ and $\tilde{\theta}$ respectively. Replacing in 3.13 and $\theta$ in $F$ with $\tilde{\alpha}$ and $\tilde{\theta}$ respectively and substituting the relation $z = E[\alpha + (E[\alpha])]$ into 3.13, we approximate the CRE as

$$CRE \approx \|PB^{-1} \tilde{E}^* E B (\mu d) + (\alpha - \tilde{\alpha}) PB^{-1} \tilde{E}^* E^* B^* (\mu d)^* + PB^{-1} \tilde{E}^* E (\mu q)\|^2$$

(13)

Thus CRE is analysed.

IV. SIMULATION RESULT

In this section, we carry out Monte-Carlo experiments to verify the performance of the proposed methods. A total of 5000 random channels for each transmit and receive antenna pair are generated in the experiments. The channel taps are complex Gaussian random variables and the channel length is $L = 65$. The variance of the channel taps is normalized by

$$\sum_{l=0}^{L-1} E\{|h_{k,l}(l)|^2\} = 1 \quad \text{for all } k, j$$

(14)

The channel noise is AWGN. The training data are QPSK symbols. The training sequences in the experiments are the optimal sequences in [9]. The size of the DFT matrix is $M = 1024$. The CP length is $L - 1 = 64$. From Fig. 5(a) and (b) it is found that our method provides good performance.

CONCLUSION

A new method for the estimation of the I/Q imbalance, by using training sequences is proposed in this paper. When only one OFDM block is available for training, our method is able to give an accurate estimate of the I/Q parameter. When two repeated OFDM blocks are available for training, a low complexity two step approach is proposed to solve the joint estimation problem. The proposed method is simple and provides good results.

REFERENCES

## Author’s Profile

<table>
<thead>
<tr>
<th>Kanchan Nikumbh</th>
<th>Achala Deshmukh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Was born in Nashik in 1989. Received the B.E. degree in Electronics and Telecommunication from Pune University, Pune in 2011. Presently pursuing M.E. in Pune University. Current research interests include MIMO communication and OFDM systems.</td>
<td>Received B.E. Degree from KBP college, Satara. Completed her M.E. in Electronics. Presently registered for Ph.D. Works as assistant professor in Sinhgad Institute of Technology. Has 19yrs of teaching experience. Current research interest are PAPR Reduction techniques for DVB-T.</td>
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